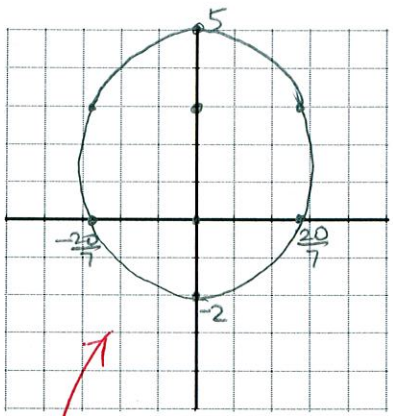
Consider the graph of the polar equation $r = \dfrac{20}{7-3\sin\theta}$. $= \dfrac{\frac{20}{7}}{1-\frac{3}{7}\sin\theta}$

SCORE: ____ / 9 PTS



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θ	r
0	$\frac{20}{7} = 2\frac{6}{7}$
$\frac{\pi}{2}$	$\frac{20}{4} = 5$
π	$\frac{20}{7} = 2\frac{6}{7}$
$\frac{3\pi}{2}$	$\frac{20}{10} = 2$

[a] Fill in the blanks.

[i] The eccentricity is $\frac{3}{7}$. (1)

[ii] The shape of the graph is a/an ELLIPSE. (1)

[iii] The equation of the directrix is $y = -\frac{20}{3}$. (1)

$ep = \frac{20}{7}$

$\frac{3}{7}p = \frac{20}{7}$

$p = \frac{20}{3}$

[iv] Find the **rectangular** coordinates of the

x – intercept(s) $(\frac{20}{7}, 0), (-\frac{20}{7}, 0)$ (1)

y – intercept(s) $(0, -2), (0, 5)$ (1)

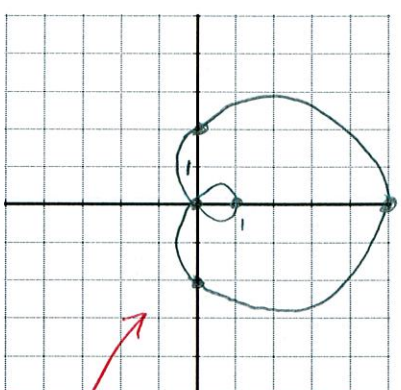
focus/foci $-2+5=3$ $(0, 3), (0, 0)$ (1)

endpoints of the latus rectum/latera recta $(\pm\frac{20}{7}, 3), (\pm\frac{20}{7}, 0)$ (1)

[b] Sketch the graph on the grid provided above. **You must provide a scale for the axes & plot all points from part [a][iv] above.**

Consider the graph of the polar equation $r = 2 + 3\cos\theta$. $0 < \left|\frac{2}{3}\right| < 1$

SCORE: ____ / 6 PTS



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θ	r
0	5
$\frac{\pi}{2}$	2
π	-1
$\frac{3\pi}{2}$	2

[a] Fill in the blanks.

[i] The shape of the graph is a/an LIMAÇON WITH LOOP. (1)

[ii] The graph DOES (1) pass through the pole.

does / does not

[iii] Find the **rectangular** coordinates of the

x – intercept(s) $(0, 0), (5, 0), (1, 0)$ (1)

y – intercept(s) $(0, 0), (0, 2), (0, -2)$ (1)

[b] Sketch the graph on the grid provided above. **You must provide a scale for the axes & plot all points from part [a][iii] above.**

Consider the graph of the polar equation $r = 2 + 2\sin 2\theta$. **SCORE: _____ / 15 PTS**

NOTE: $(r, \pi-\theta)$ and $(-r, \pi-\theta)$ tests do NOT show that the graph is symmetric

$\theta = \frac{\pi}{2}$ POLAR AXIS

[a] **Using the information above, and the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis, and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

$\theta = \frac{\pi}{2}$: $-r = 2 + 2\sin 2(-\theta)$

$-r = 2 - 2\sin 2\theta$

(1) $r = -2 + 2\sin 2\theta$ ×

POLAR AXIS: $r = 2 + 2\sin 2(-\theta)$

(1) $r = 2 - 2\sin 2\theta$ ×

POLE: $r = 2 + 2\sin 2(\pi + \theta)$

$r = 2 + 2\sin(2\pi + 2\theta)$

$r = 2 + 2[\sin 2\pi \cos 2\theta + \cos 2\pi \sin 2\theta]$

(1) $r = 2 + 2\sin 2\theta$ ✓

"NO" AND "NOT SYMMETRIC" ARE INCORRECT

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION
Over the pole	SYMMETRIC

(2) IF ALL 3 CORRECT

(1) IF 2 OF 3 CORRECT

(0) IF NONE OR ONLY 1 CORRECT

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot first (before using reflections to draw the rest of the graph) ?

$[0, \pi]$ OR $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (1) EITHER ANSWER IS OK

[c] Find the angles in the minimum interval in part [b] at which the graph goes through the pole.

$0 = 2 + 2\sin 2\theta$

(1) $\sin 2\theta = -1$

IF USING $[0, \pi]$: $0 \le 2\theta \le 2\pi$, $2\theta = \frac{3\pi}{2}$, $\theta = \frac{3\pi}{4}$

OR

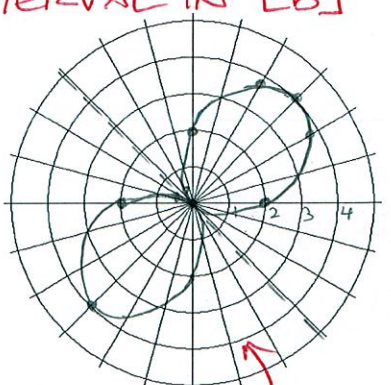
IF USING $[-\frac{\pi}{2}, \frac{\pi}{2}]$: $-\pi \le 2\theta \le \pi$, $2\theta = -\frac{\pi}{2}$, $\theta = -\frac{\pi}{4}$

(1) MUST MATCH YOUR INTERVAL IN [b]

[d] Find the value of r for all common angles in the minimum interval in part [b].

θ	r
0	(½) 2
$\frac{\pi}{6}$	(½) $2+\sqrt{3} \approx 3.8$
$\frac{\pi}{4}$	(½) 4
$\frac{\pi}{3}$	(½) $2+\sqrt{3} \approx 3.8$
$\frac{\pi}{2}$	(½) 2

θ	r
$-\frac{\pi}{3}$ or $\frac{2\pi}{3}$	(½) $2-\sqrt{3} \approx 0.2$
$-\frac{\pi}{4}$ or $\frac{3\pi}{4}$	0
$-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	(½) $2-\sqrt{3} \approx 0.2$
$-\frac{\pi}{2}$ or π	(½) 2



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[d] Sketch the graph on the grid provided below. **You must provide a scale for the polar axis & plot all points from part [c] above.**